

German University in Cairo
Computer Science and Engineering Department
Prof. Dr. Slim Abdennadher

November, 2010

Constraint Programming, Spring term 2010

Exam

Bar Code

Instructions: Read carefully before proceeding.

Good Luck!

Don't write anything below ;-)

Exercise 1

Given the following dinner problem:

We are going out to dinner taking 1-6 grandparents, 1-10 parents and/or 1-40 children. Grandparents cost 3 dollars for dinner, parents 2 dollars and children 0.50 dollars. There must be 20 total people at dinner and it must cost 20 dollars.

The problem to be solved is to find how many grandparents, parents and children are going to dinner.

Model the problem as a constraint problem in CLPFD!

Solution:

```
solve([Grandparents, Parents, Children]) :-
    Grandparents in 0..100,
    Parents       in 0..100,
    Children      in 0..100,
    Grandparents * 3 + Parents * 2 + Children / 2 #= 20,

    Grandparents + Parents + Children #= 20, % number of people = 20

    % must be some of each
    Grandparents #> 0,
    Parents       #> 0,
    Children      #> 0,

    labeling([], [Grandparents, Parents, Children]).
```

Exercise 2

A magic sequence of length n is a sequence of integers $(x_0, x_1, \dots, x_{n-1})$ such that when the variable x_i has the value n_j then the sequence contains exactly n_j i 's, i.e. for all i in 0 to $n-1$, the number i occurs exactly x_i times in the sequence.

For instance, 6,2,1,0,0,0,1,0,0,0 is a magic sequence since 0 occurs 6 times in it, 1 occurs twice, 2 occurs once,

1,2,1,0 is a magic sequence too!

Your task is to find a basic mathematical model for finding magic sequences.

Solution:

- Variables and Domains:
Magic sequence items: $x_0, \dots, x_k \in \{0, \dots, k\}$.
- Constraints: Value of x_i is i times in the sequence:

$$\forall i \in \{0, \dots, k-1\} x_i = \sum_{x_j=i} 1$$

Exercise 3

- a) What do arc-consistency algorithms do? What are the aims of arc-consistency algorithms?

Solution:

the arc consistency algorithms repeatedly restrict the domains of the variables until the property hold true. That is all the arcs are made consistent. They examine each arc in the constraint graphs in turn, and remove those values of the first node that are not consistent with any values of the second variable. If any value from the domain of one variable do not have any value of another variable which satisfies the constraint, these values are removed.

The aim of the arc consistency algorithms are that the size of the CSP is reduced so as easier to solve. The earlier this is made, the smaller the size of the search tree is, thus the more efficient the search is to find solutions quickly. The reduced CSPs after applying arc-consistency should be equivalent to the original one. That is, no solutions should be ruled out by the algorithms.

- b) In the lectures two algorithms for arc-consistency were introduced. A general one and an algorithm called AC-3. What is the difference between those algorithms?

Solution:

The general one checks all constraints whenever any values are removed from the domain of any variables in the previous iteration. This includes those even not affected by the removal of values in the previous iteration.

AC-3 only checks those variables which are affected by the variables whose domains are changed by the removal of values in previous iterations. (2 marks) Thus, AC-3 is more efficient than the general arc-consistency algorithm.

Exercise 4

Consider the following CSP on the domains $D(X) = D(Y) = D(Z) = [1, 2]$

$$X \neq Y \wedge Y \neq Z \wedge X \neq Z$$

- a) Is the CSP arc-consistent. If no, transform the CSP to an equivalent CSP which is arc-consistent. Show your workout.

Solution:

It is arc-consistent

- b) Path consistency is a property similar to arc consistency, but considers pairs of variables instead of only one. A pair of variables is path-consistent with a third variable if each consistent evaluation of the pair can be extended to the other variable in such a way that all binary constraints are satisfied. Formally, x_i and x_j are path consistent with x_k if, for every pair of values (a, b) that satisfies the binary constraint between x_i and x_j , there exists a value c in the domain of x_k such that (a, c) and (b, c) satisfy the constraint between x_i and x_k and between x_j and x_k , respectively.

Determine whether the above CSP is path-consistent or not. Show your workout.

Solution:

It is not path-consistent!